Critical behavior of elastic and transport properties of graphene

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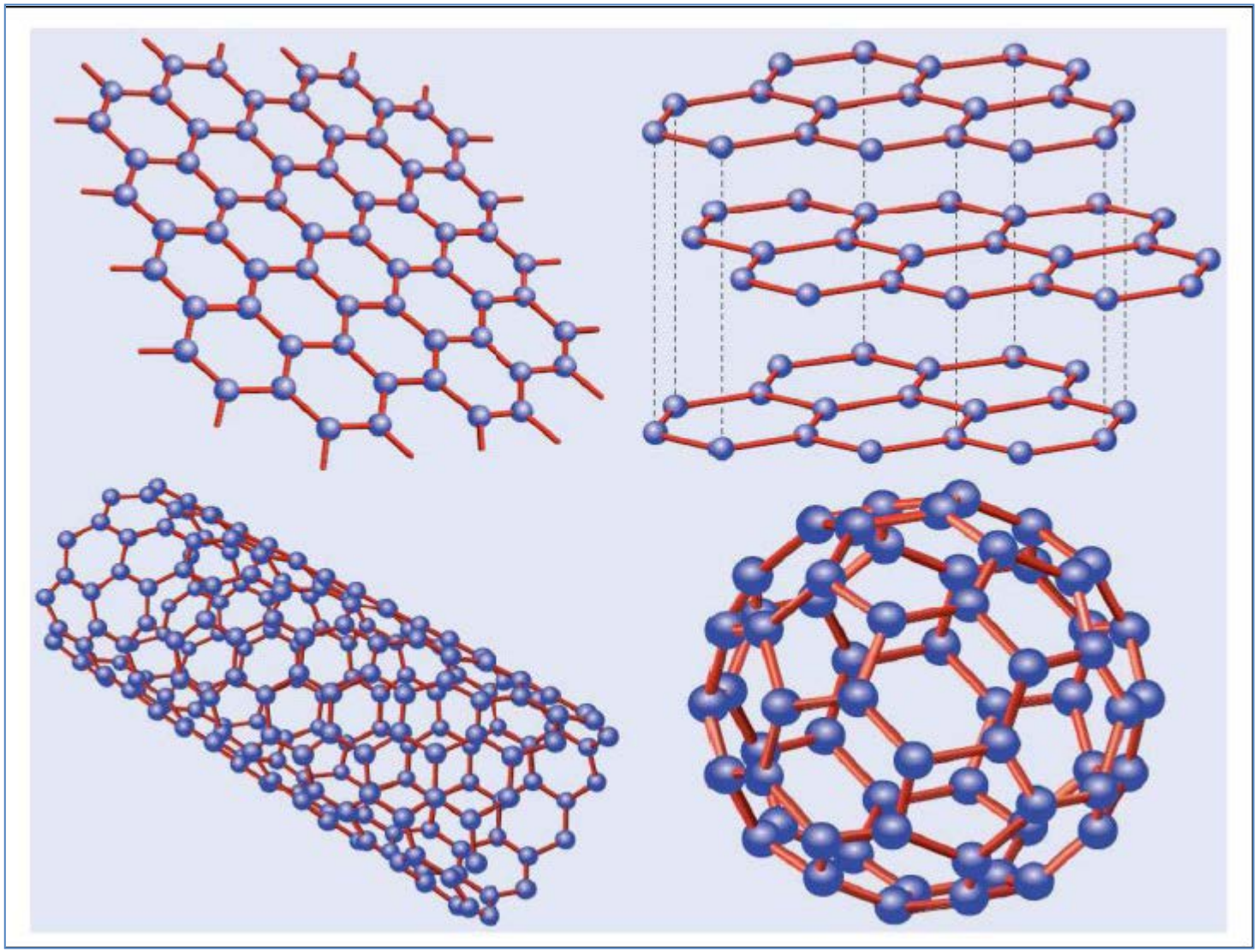
Outline

- Introduction. Unusual properties of graphene
- Suspended graphene as elastic membrane. Flexural phonons (FP), ripples , huge thermal fluctuations
- Crumpling transition. Renormalization of bending rigidity. Stability of graphene
- Quasielastic scattering by FP. Drude conductivity in the Dirac point. Power-law scaling with temperature
- Effect of electron-electron (ee) interaction. 1) Velocity relaxation ➔ finite resistivity 2) Screening of FP
- Competition between ee and FP. Transport regimes in the Dirac point. Away from Dirac point: "metallic" ➔ "insulating " T-dependence
- Effect of disorder on crumpling transition in graphene. A mechanism of ripples formation

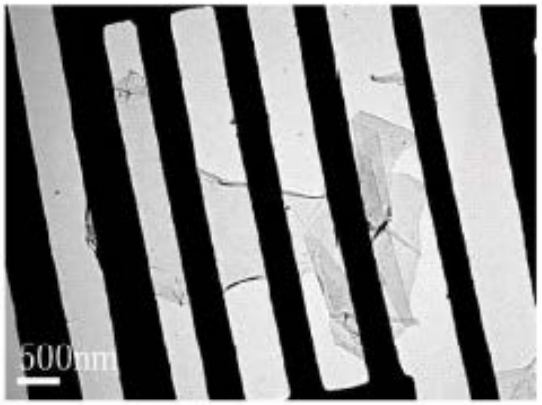
Graphene: monoatomic layer of carbon



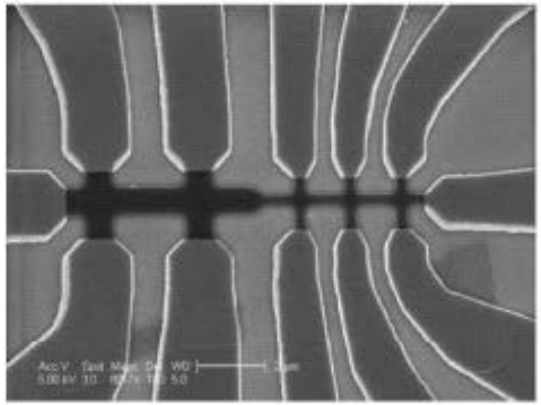
First isolated and explored: Manchester (Geim, Novoselov, et al., 2004)

Nobel Prize 2010 (Andre Geim & Konstantin Novoselov)

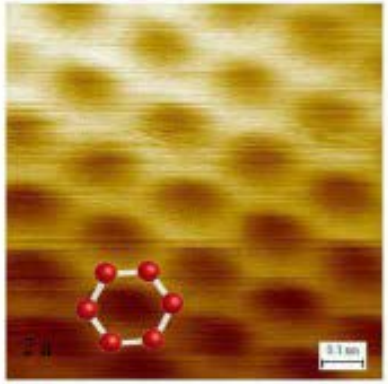
Graphene samples



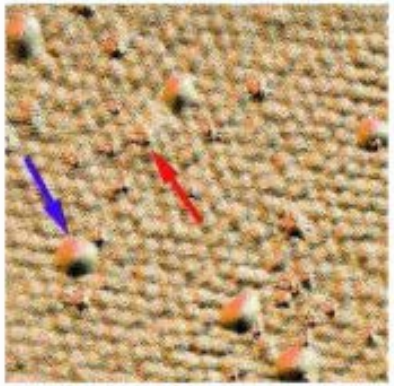
Suspended sample



Hall bar



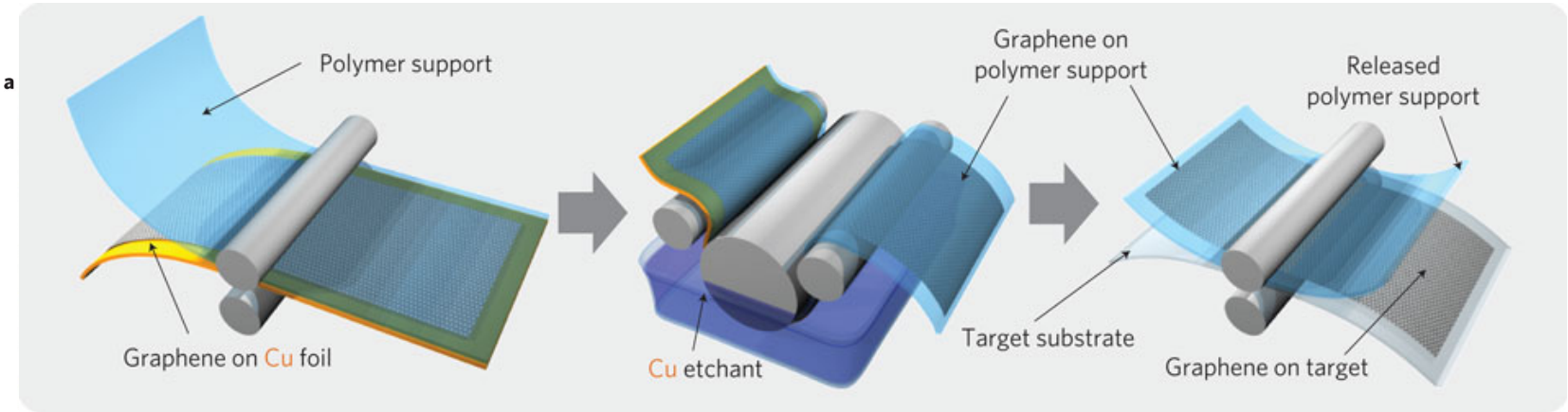
Micro-mechanical cleavage

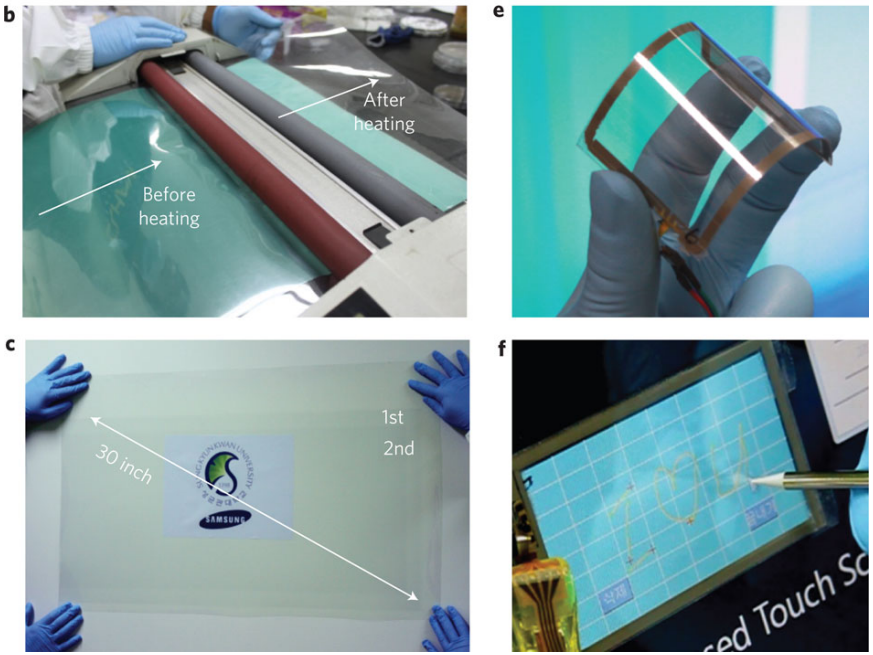


Epitaxial growth

carrier mobility: up to ~20,000 cm²/V·s at 300K; ~200,000 cm²/V·s at 4K

Progress in manufacturing graphene





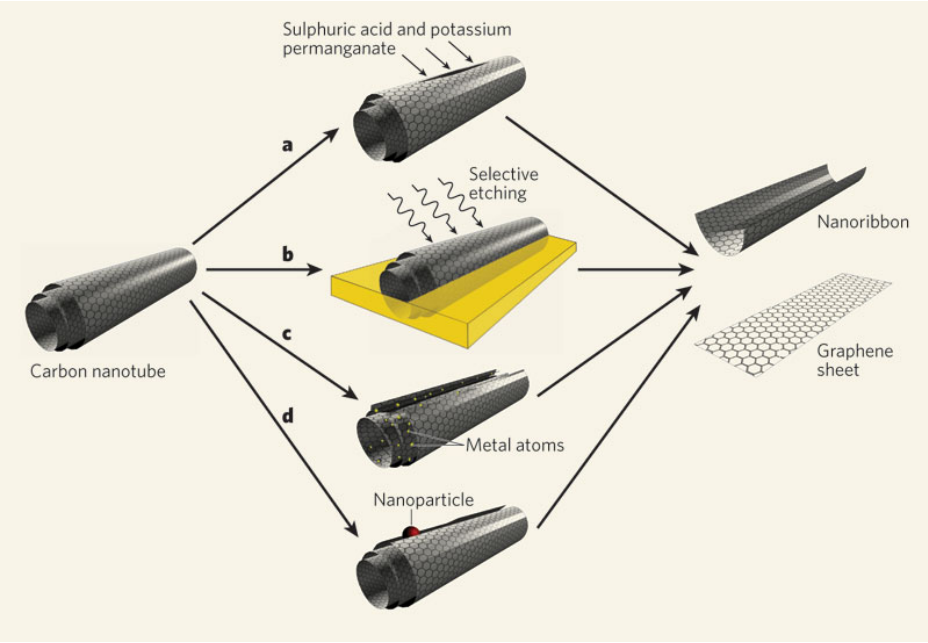
Roll-to-roll production of **30-inch** graphene films for transparent electrodes

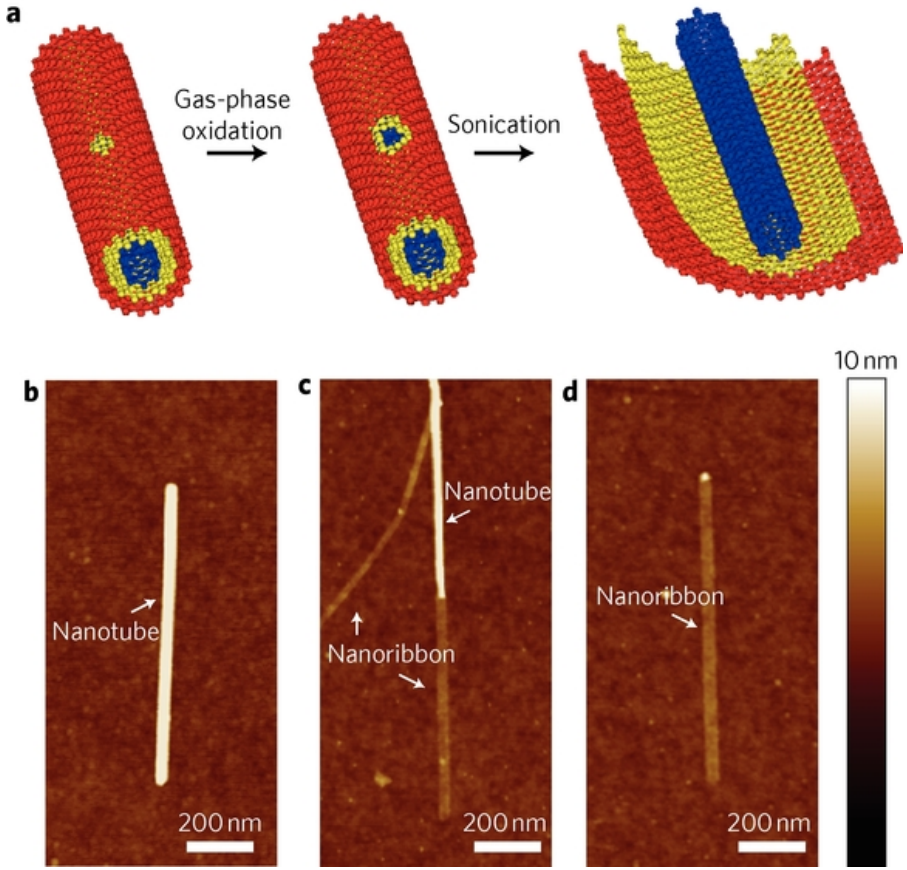
Bae et al. *(Korea-Japan collaboration),*

Monolayer graphene films grown by chemical vapour deposition: QHE, low resistance, high **transparency**

Progress in manufacturing graphene

Unzipping nanotubes: producing high quality nanoribbons

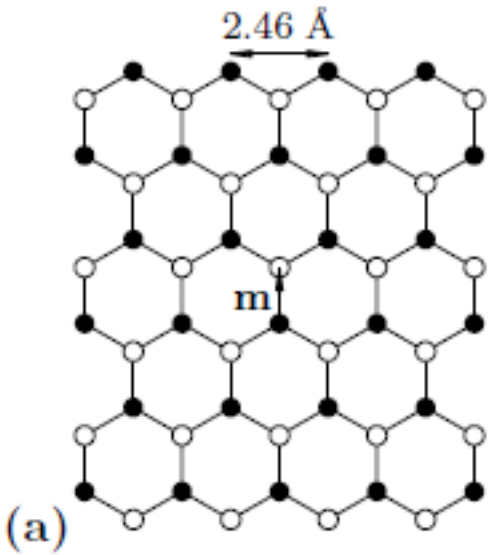


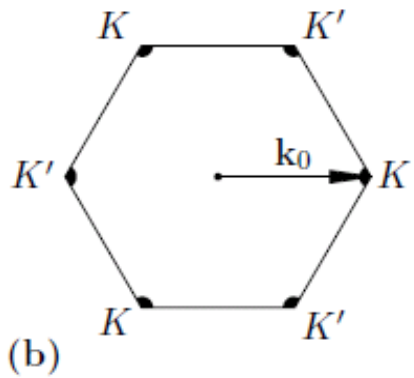


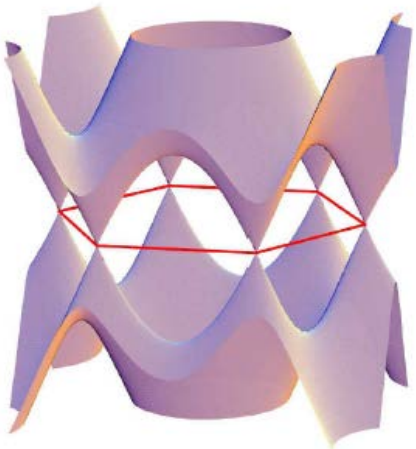
Jiao et al. (Stanford CA)

+ other groups

From tight-binding approximation to Dirac fermions





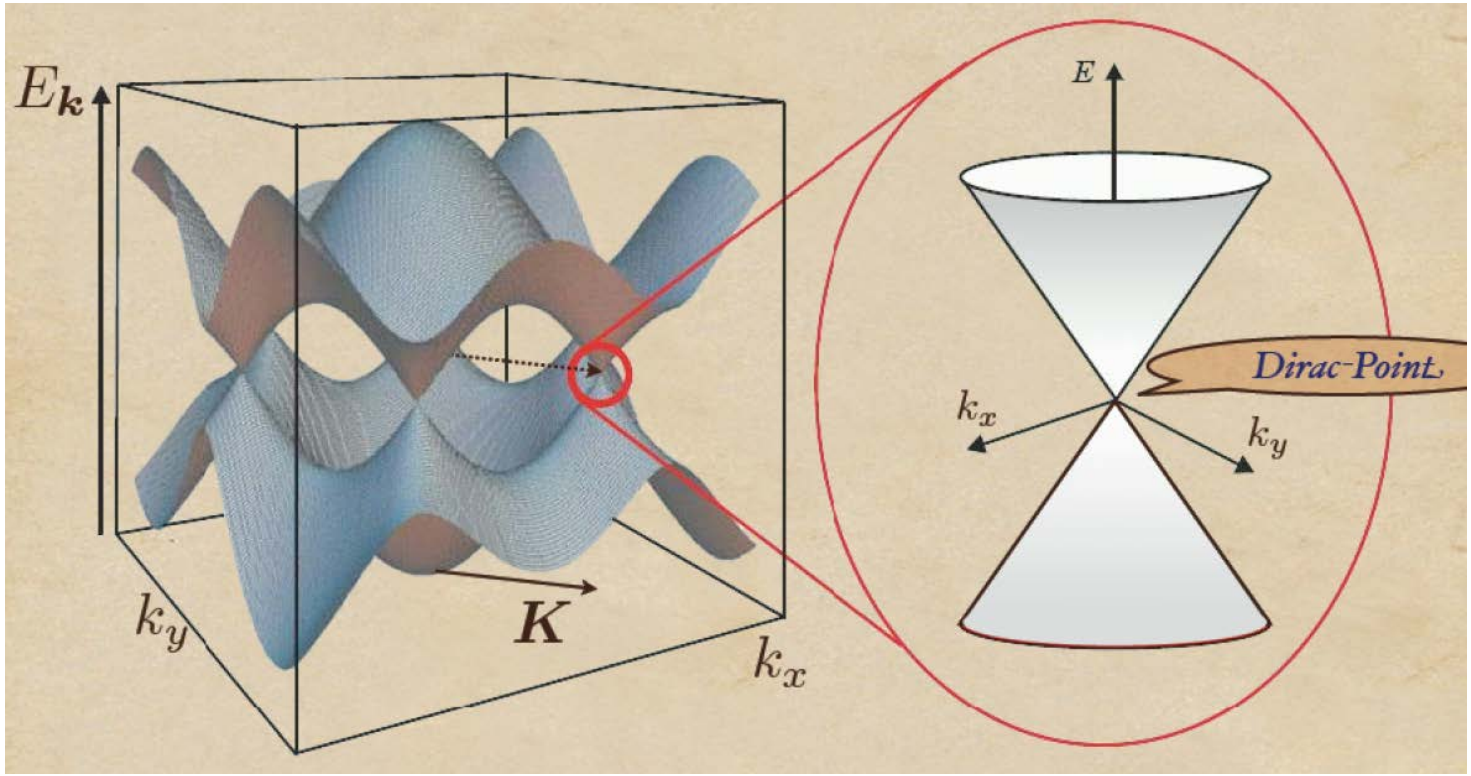


- two sublattices: A, B (σ Pauli matrices)
- two valleys: K, K′ (τ Pauli matrices)
- massless Dirac Hamiltonian:

K: $H = v_0 \boldsymbol{\sigma} \mathbf{p}$ $\quad$ K': $H = -v_0 \boldsymbol{\sigma}^T \mathbf{p}$

$$\boldsymbol{\sigma} = \{\sigma_x, \sigma_y\}$$

Clean graphene: band structure



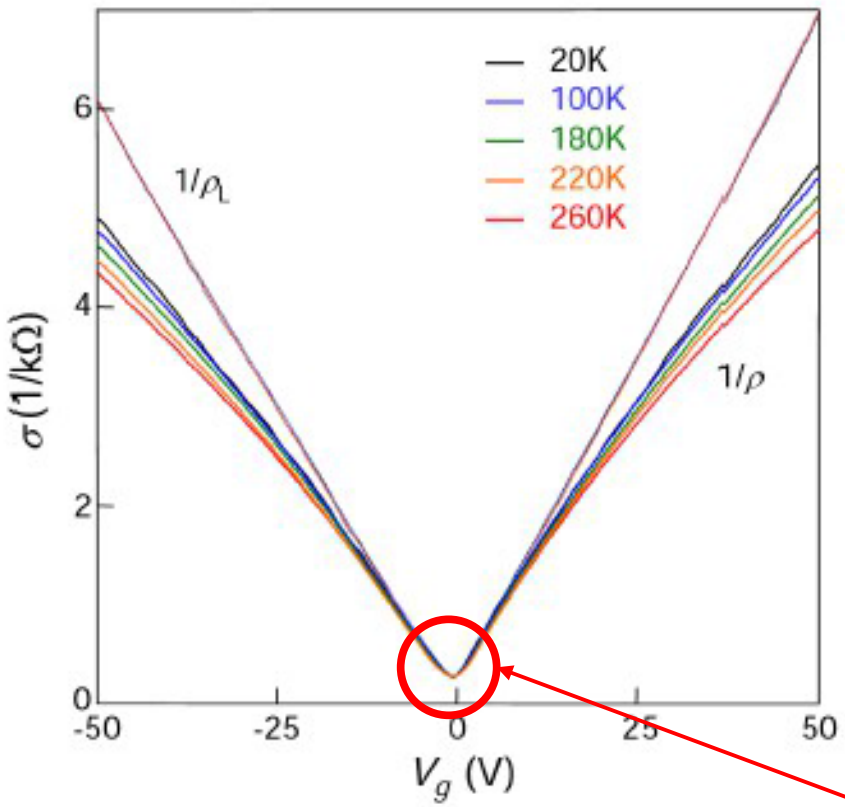
Near the Dirac point:

$$H = \hbar v \boldsymbol{\sigma} \mathbf{k} \qquad \epsilon_\alpha(\mathbf{k}) = \alpha \hbar v k,$$

$$\psi_{\mathbf{k}\alpha} = e^{i\mathbf{k}\mathbf{r}} |\chi^\alpha_{\mathbf{k}}\rangle, \qquad |\chi^\alpha_{\mathbf{k}}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\varphi_{\mathbf{k}}/2} \\ \alpha e^{i\varphi_{\mathbf{k}}/2} \end{pmatrix}$$

$$\alpha = \pm$$

Density dependence of conductivity: graphene on substrate



Linear density dependence away from charge neutrality:

* Long-range Coulomb impurities

* Ripples

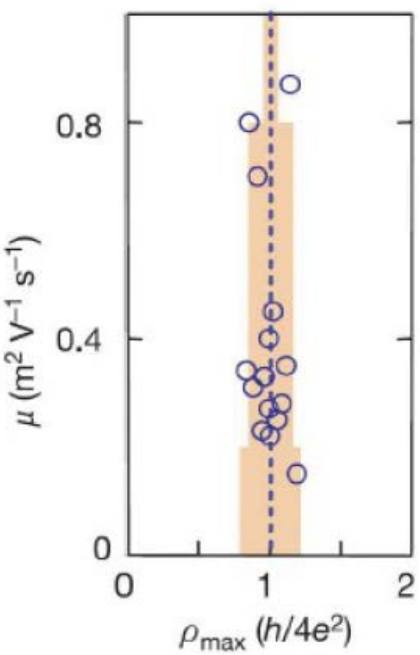
* Resonant scatterers

Ostrovsky, Gornyi, Mirlin '06

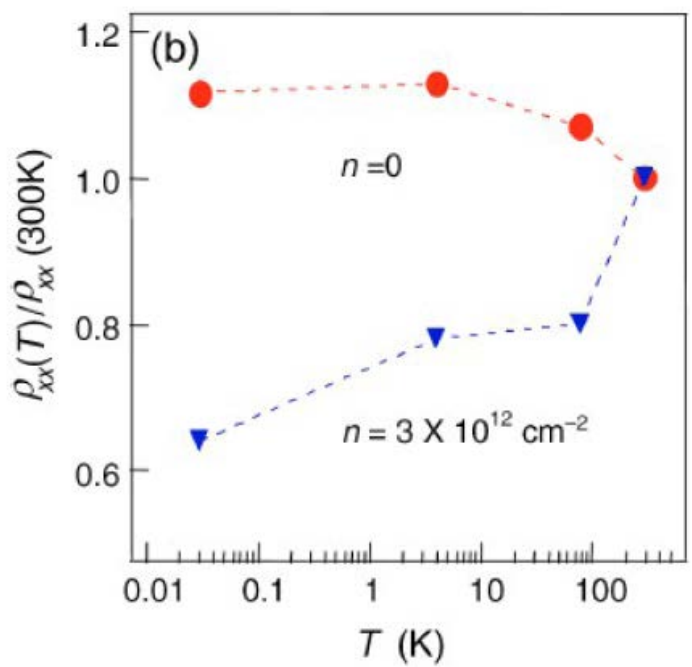
Minimal conductivity

Manchester (Geim, Novoselov et al.) '05

Minimal conductivity



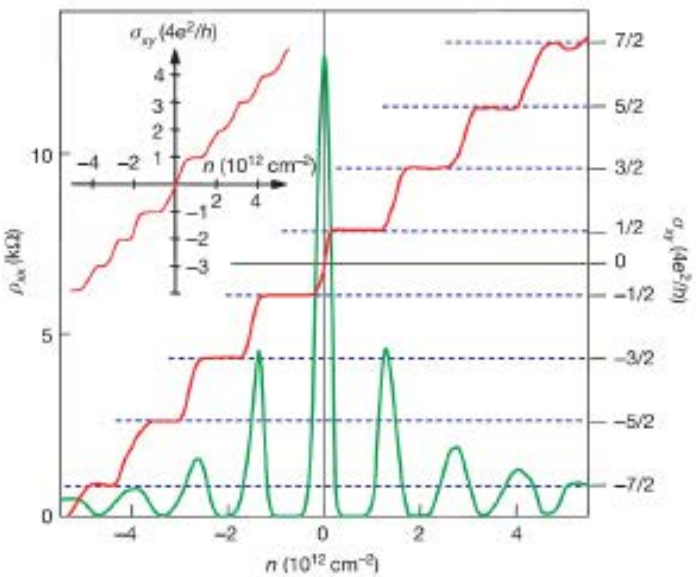
Manchester '05



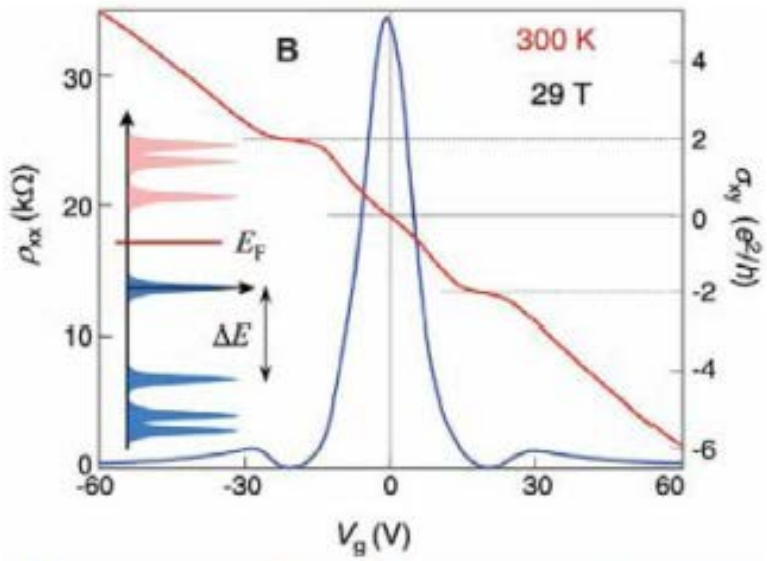
Columbia '07

- of order e^2/h
- temperature independent $\Longrightarrow$ no localization!

Quantum Hall effect in graphene



Novoselov, Geim et al '05

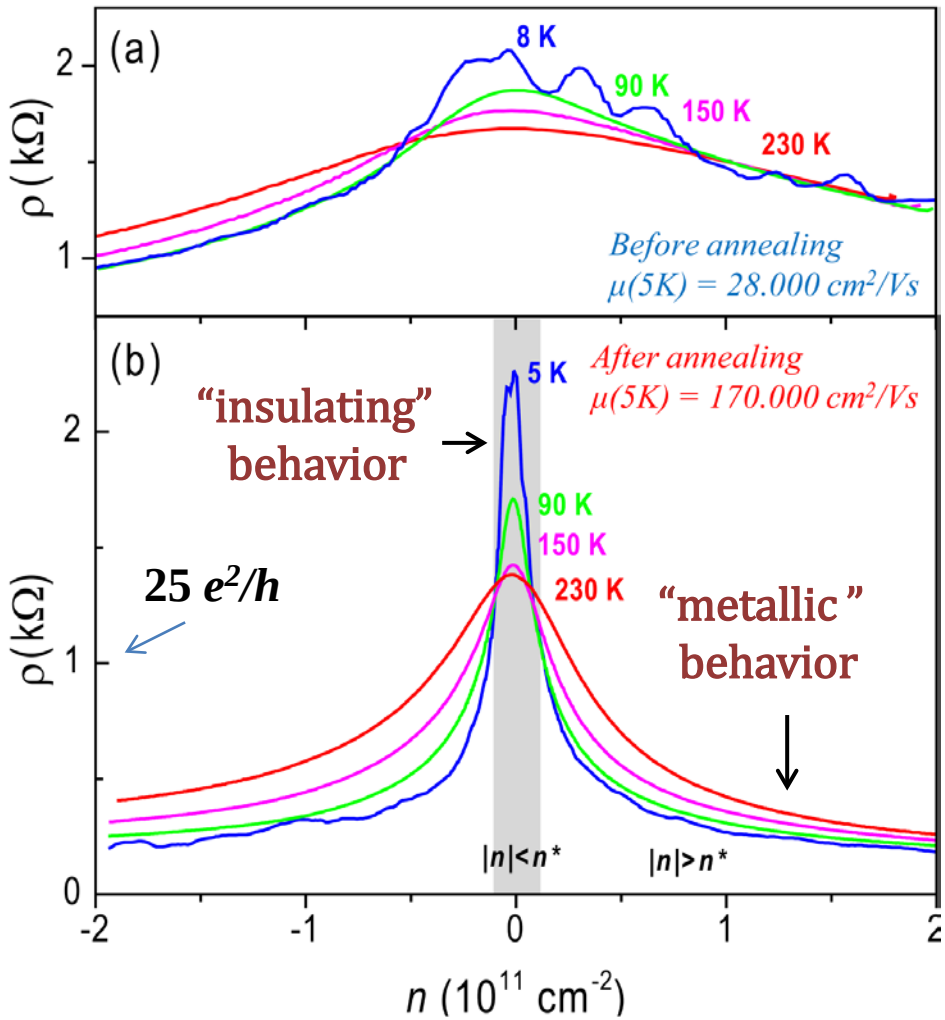


Novoselov, Geim, Stormer, Kim '07

Anomalous quantum Hall effect

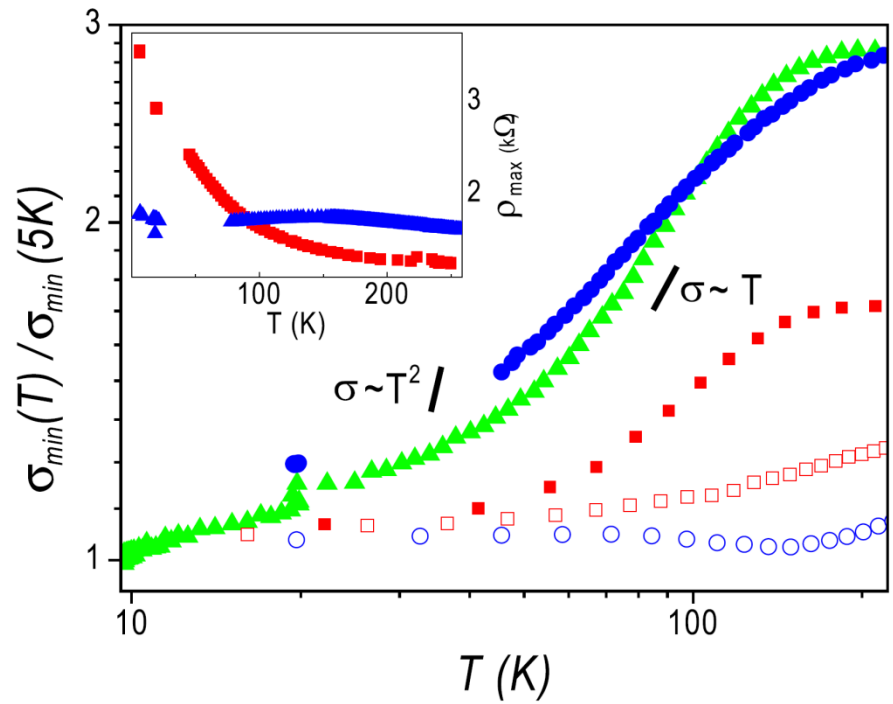
- only odd plateaus: $\sigma_{xy} = (2n+1)2e^2/h$
- QHE transition at zero concentration
- visible up to room temperature!

Density dependence of conductivity: suspended graphene



Temperature dependence of minimal conductivity:

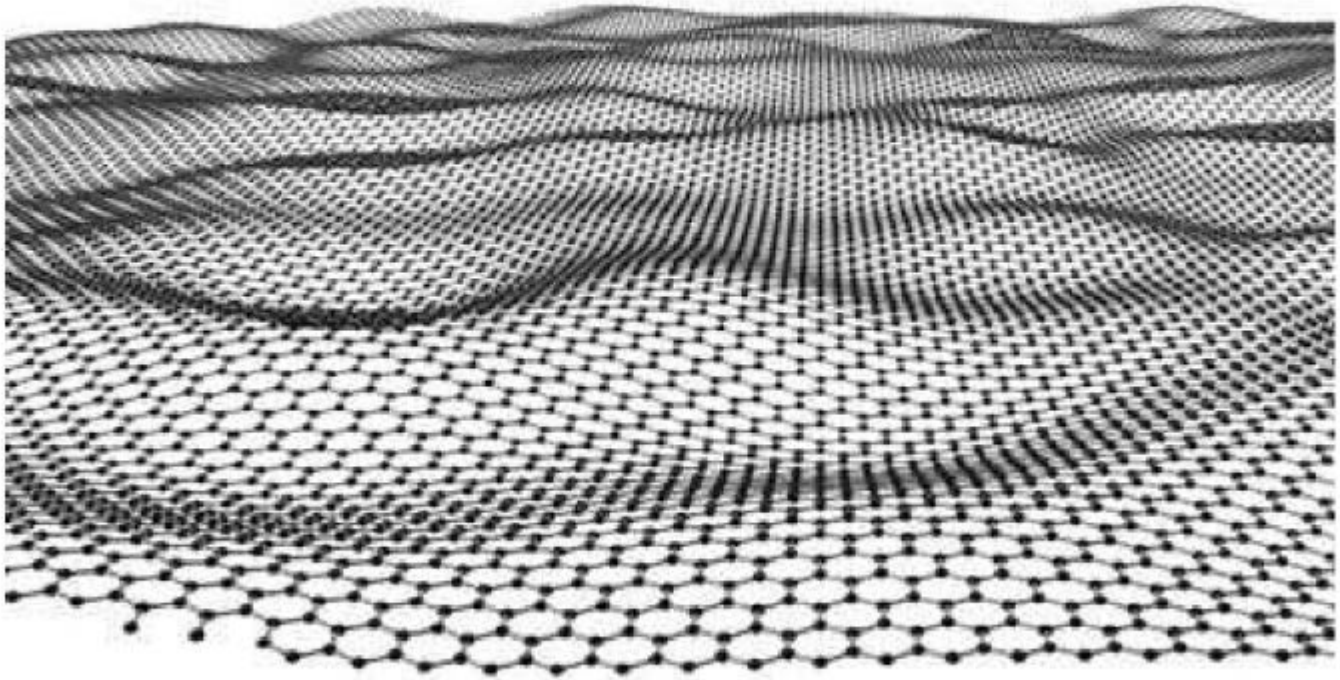
Weaker disorder; flexural phonons & Coulomb interaction



K. Bolotin, K.Sikes, J.Hone, H.Stormer, P.Kim, PRL (2008)

Suspended graphene: flexural phonons

Ripples = „snapshot“ of flexural phonons



Meyer, Geim, Katsnelson, Novoselov, Booth, Roth, Nature'07

Graphene as elastic membrane

Elastic energy

$$E = \frac{1}{2}\int d\mathbf{r}\left[\rho(\dot{\mathbf{u}}^2 + \dot{h}^2) + \varkappa(\Delta h)^2 + 2\mu u_{ij}^2 + \lambda u_{kk}^2\right]$$

$\mathbf{u}(\mathbf{r}), h(\mathbf{r})$ are in-plane and out-of-plane distortions

$$u_{ij} = \frac{1}{2}[\partial_i u_j + \partial_j u_i + (\partial_i h)(\partial_j h)]$$

Strain tensor

$\rho \simeq 7.6 \cdot 10^{-7}\mathrm{kg/m}^2$	mass density of graphene
$\lambda \simeq 3\mathrm{eV/\AA}^2 \quad \mu \simeq 3\mathrm{eV/\AA}^2$	elastic constants
$\varkappa \approx 1\mathrm{eV}$	bending rigidity

$\omega_{\parallel\mathbf{q}} = s_\parallel q \ , \quad \omega_{\perp\mathbf{q}} = s_\perp q$	in-plane phonons
$s_\parallel = [(2\mu+\lambda)/\rho]^{1/2} \simeq 2\cdot 10^6\,\mathrm{cm/s}, \quad s_\perp = (\mu/\rho)^{1/2} \simeq 1.3\cdot 10^6\,\mathrm{cm/s}$	

Flexural phonons (FP)

$$E_\perp = \frac{1}{2}\int d\mathbf{r}\left[\rho\dot{h}^2 + \kappa(\Delta h)^2\right]$$

$$h(\mathbf{r}) = \sum_{\mathbf{q}} \sqrt{\frac{\hbar}{2\rho\omega_{\mathbf{q}}S}}(b_{\mathbf{q}} + b^\dagger_{-\mathbf{q}})e^{i\mathbf{q}\mathbf{r}}$$

out-of-plane flexural mode

$$\omega_q = Dq^2$$

soft dispersion of FP

$$D = \sqrt{\kappa/\rho} = 0.46 \cdot 10^{-2} cm^2/s$$

Лирическое отступление – фраза из Википедии:

.....Графен не удавалось создать до 2005 года. Кроме того, ещё раньше было **доказано теоретически**, что свободную идеальную двумерную плёнку получить **невозможно** из-за нестабильности относительно сворачивания или скручивания [1]. Тепловые флуктуации приводят к плавлению двумерного кристалла при любой конечной температуре.....

[1] *Ландау Л. Д., Лифшиц Е. М.* Статистическая физика. — 2001

????????

Thermal fluctuations

$$b_{\mathbf{q}} = \sqrt{N_{\mathbf{q}}} e^{-i\varphi_{\mathbf{q}}}$$

$$N_{\mathbf{q}} \approx \sqrt{T/\hbar\omega_{\mathbf{q}}} \gg 1$$

$$h(\mathbf{r}) = \sum_{\mathbf{q}} \sqrt{\frac{2T}{\varkappa q^4 S}} \cos(\mathbf{qr} + \varphi_{\mathbf{q}})$$

$$G(\mathbf{q}) = \langle h_{\mathbf{q}} h^*_{\mathbf{q}} \rangle = \frac{T}{\varkappa q^4}$$

correlation function of FP

$$\sqrt{\langle h^2(\mathbf{r}) \rangle} \propto \sqrt{\frac{T}{\varkappa} \int \frac{d^2\mathbf{q}}{q^4}} \propto \sqrt{\frac{T}{\varkappa}} L$$

for graphene at room temperature: $\sqrt{T/\varkappa} \approx 0.2$

Thermal fluctuations with small q are huge !!!!!

Quasielastic scattering by FP

$$V_{e,ph} = V + V_{\mathbf{A}} = g_1 u_{ii} + g_2 \boldsymbol{\sigma}\mathbf{A}$$
$$A_x = 2u_{xy}, \quad A_y = u_{xx} - u_{yy}$$

$$V = g_1 (\nabla h)^2/2$$

FP contribution to the deformation potential

$g_1 \simeq 30$ eV deformation coupling constant,
$g_2 \simeq 1.5$ eV coupling to gauge field

$$V(\mathbf{r}) = \frac{g_1 T}{\varkappa S} \sum_{\mathbf{q}_1,\mathbf{q}_2} \frac{\mathbf{q}_1\mathbf{q}_2}{q_1^2 q_2^2} \sin(\mathbf{q}_1\mathbf{r} + \varphi_{\mathbf{q}_1}) \sin(\mathbf{q}_2\mathbf{r} + \varphi_{\mathbf{q}_2})$$

Golden rule calculation

Theory:

$$\sigma_{\rm ph} = \frac{e^2}{\hbar} \frac{\pi^2 N}{24 g^2 \ln(q_T L)} \approx 10^{-3} \frac{e^2}{h}$$

Theory yields unrealistic (too small) values of conductivity !!!

Experiment:

$$\sigma_{\rm ph} \sim 10 \div 50 \frac{e^2}{h}$$

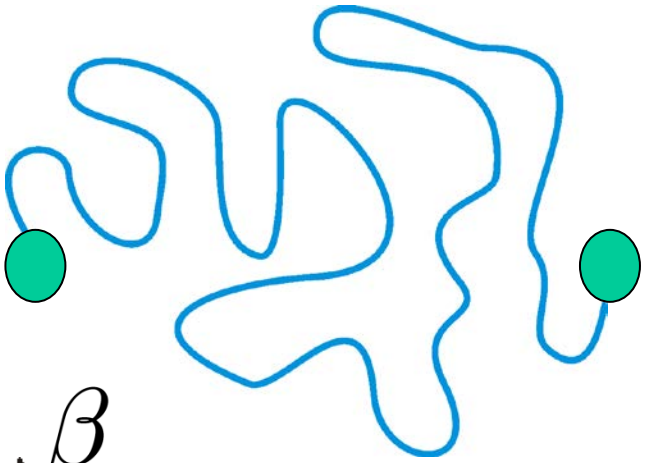
$$g = \frac{g_1}{\sqrt{32}\varkappa} \simeq 5.3$$

dimensionless coupling constant

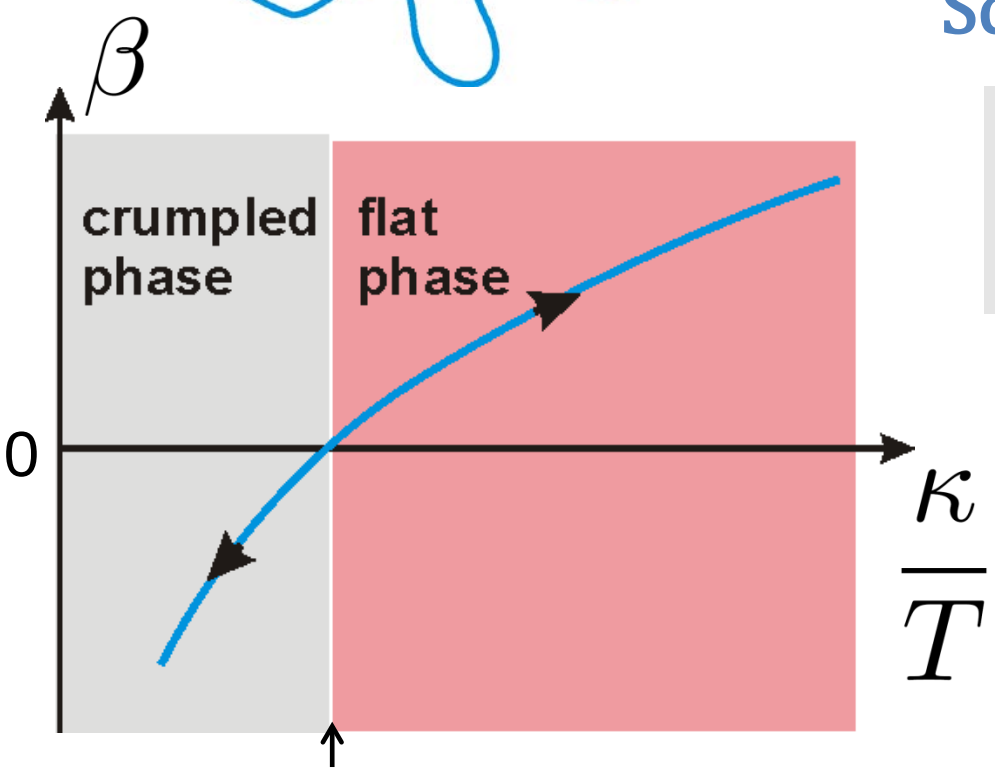
$N = 4$ spin×valleys,
$q_T = T/\hbar v$

Crumpling transition of membrane: key parameter κ/T

Crumpled phase, $\kappa/T \rightarrow 0$



Flat phase, $\kappa/T \rightarrow \infty$



crumpling phase transition

Scaling of bending rigidity

$$\frac{d\ln(\kappa/T)}{d\ln L} = \beta(\kappa/T)$$

D. Nelson, T. Piran, S. Weinberg *Statistical Mechanics of Membranes and Surfaces* (1989).

Physics behind: anharmonic coupling with in-plane modes

For graphene $\kappa/T \approx 30$ even for T=300 K ➔ flat phase

Bending rigidity increases with increasing the system size (or decreasing the wave vector) :

$$\kappa \sim L^{\eta}, \quad q^{-\eta}$$

η - critical exponent (≈ 0.7)

F.David and E. Guitter, Europhys. Lett. (1988)

P. Le Doussal and L. Radzihovsky, PRL (1992)

$$\beta \to \eta, \quad \text{for} \quad \kappa/T \to \infty$$

$$\frac{h}{L} \sim \frac{1}{L^{\eta/2}}$$

in the thermodynamic limit fluctuations are suppressed

$$\omega \sim q^{2-\eta/2}$$

dispersion is modified

critical behavior of elastic properties

Theory of crumpling transition

$$F = \int d^D x \left\{ \frac{\varkappa_0}{2}(\partial_\alpha \partial_\alpha \mathbf{R})^2 - \frac{t}{2}(\partial_\alpha \mathbf{R} \partial_\alpha \mathbf{R}) + u(\partial_\alpha \mathbf{R} \partial_\beta \mathbf{R})^2 + v(\partial_\alpha \mathbf{R} \partial_\alpha \mathbf{R})^2 \right\}$$

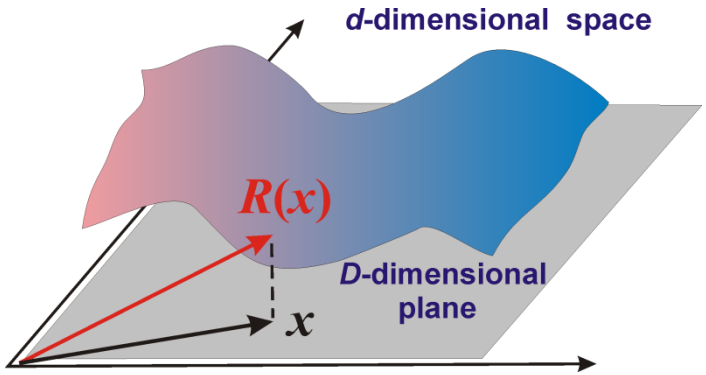
$$\alpha, \beta = 1, \ldots, D$$

Paczuski, Kardar, Nelson , PRL,1988

$\mathbf{R}(\mathbf{x})$ is d-dimensional vector

$\mathbf{x}$ is D-dimensional vector

For physical membranes d=3, D=2



Mean field ➡ $\mathbf{R} = \xi \mathbf{x}$ ➡ $F = -\xi^2 t + 2\xi^4 (u + Dv)$

$$\partial F / \partial \xi = 0 \Rightarrow \xi^2 = \begin{cases} \dfrac{t}{4(u + Dv)}, & \text{for } t > 0 \\ 0, & \text{for } t < 0 \end{cases}$$

flat phase

crumpled phase

$t \propto T_c - T$ ➡ $\xi^2 \propto T_c - T$

Flat phase ($T<T_c$, $\xi>0$)

$$\mathbf{R} = \xi \mathbf{r} \qquad \mathbf{r} = \mathbf{x} + \underbrace{\mathbf{u} + \mathbf{h}}$$

in-plane and out-of-plane fluctuations

$$\mathbf{u} = (u_1, ..., u_D), \ \mathbf{h} = h_1, ..., h_{d-D}$$

$$F = \int d^D x \left\{ \frac{\varkappa}{2}(\Delta \mathbf{r})^2 + \frac{\mu}{4}(\partial_\alpha \mathbf{r} \partial_\beta \mathbf{r} - \delta_{\alpha\beta})^2 + \frac{\lambda}{8}(\partial_\alpha \mathbf{r} \partial_\alpha \mathbf{r} - D)^2 \right\}$$

$$\varkappa = \varkappa_0 \xi^2, \ \mu = 4u\xi^4, \ \lambda = 8v\xi^4$$

$$\mu, \lambda \propto (T_c - T)^2, \quad \kappa \propto T_c - T$$

Elastic constants turn to zero in the transition point

Strain tensor

$$u_{\alpha\beta} = \frac{1}{2}\left(\partial_\alpha \mathbf{r}\partial_\beta \mathbf{r} - \delta_{\alpha\beta}\right) \approx \frac{1}{2}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha + \partial_\alpha \mathbf{h}\partial_\beta \mathbf{h}\right)$$

$$F = \int d^D x \left\{ \frac{\varkappa}{2}(\Delta \mathbf{h})^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}$$

Renormalization of elastic constants

$$d \to \infty, \quad (1/d) - \text{expansion}$$

David, Guitter, Europhys. Lett. (1988), Radzihovsky, Le Doussal, J.Phys. (Paris) (1991)

Hubbard – Stratonovich transformation

➡ **decouples $(\partial r)^4$ terms**

$$e^{-F(\mathbf{r})/T} = \int \{d\chi_{\alpha\beta}\}\, e^{-\int d^D\mathbf{x}\left\{\frac{\varkappa d}{2T}(\Delta\mathbf{r})^2 + \frac{id}{2}\chi_{\alpha\beta}(\partial_\alpha\mathbf{r}\partial_\beta\mathbf{r} - \delta_{\alpha\beta}) - \frac{Td}{4\mu}\left(\chi^2_{\alpha\beta} - \frac{\lambda}{2\mu+\lambda D}\chi^2_{\alpha\alpha}\right)\right\}}$$

$$\mathbf{r} = \xi\mathbf{x} + \delta\mathbf{r}$$

$$\int \{d\delta\mathbf{r}\} e^{-F(\mathbf{r})/T} = e^{-\int d^D\mathbf{x}\left\{\ln\det\hat{M} - \frac{id}{2}\chi_{\alpha\beta}\delta_{\alpha\beta} - \frac{Td}{4\mu}\left(\chi^2_{\alpha\beta} - \frac{\lambda}{2\mu+\lambda D}\chi^2_{\alpha\alpha}\right)\right\}}$$

$$\hat{M} = -\varkappa\Delta^2 + iT\partial_\alpha\chi_{\alpha\beta}\partial_\beta$$

First, we look for homogeneous solution for χ:

$$\chi_{\alpha\beta} = -i\chi\delta_{\alpha\beta} \quad \Rightarrow \quad \ln\det\hat{M} = \int_0^\Lambda \frac{d^D\mathbf{k}}{(2\pi)^D}\ln\left(\varkappa k^4 + T\chi k^2\right)$$

Saddle-point equations

$$F_{eff} \propto \chi(1-\xi^2) + \frac{T\chi^2}{2\mu+\lambda D} - \frac{1}{D}\int_0^\Lambda \frac{d^D\mathbf{k}}{(2\pi)^D} \ln\left(\varkappa k^4 + T\chi k^2\right)$$

$$\frac{\partial F_{eff}}{\partial \chi} = 0 \Rightarrow \quad 1-\xi^2 + \chi\frac{2T}{2\mu+\lambda D} = \frac{T}{D}\underbrace{\int \frac{d^D k}{(2\pi)^D}\,\frac{1}{\varkappa k^2 + \chi T}}_{\text{logarithmically diverge for } D=2}$$

$$\frac{\partial F_{eff}}{\partial \xi} = 0 \Rightarrow \quad \xi\chi = 0$$

In the flat phase: $\xi \neq 0 \Rightarrow \chi = 0$

D=2 → $$\frac{d\xi^2}{d\Lambda} = -\frac{T}{4\pi\varkappa}$$

$\Lambda = \ln(L/a)$

thermal fluctuations

$\xi \to 0$, for certain value of L

Within this approximation flat phase is destroyed by thermal fluctuations

Renormalization of bending rigidity

David, Guitter, Europhys. Lett. (1988), Le Doussal, Radzihovsky, PRL (1992)

$$F = \int d^D x \left\{ \frac{\varkappa}{2} (\Delta \mathbf{h})^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}$$

$$G_{ij} = \langle h_i(\mathbf{q}) h_j(-\mathbf{q}) \rangle = \frac{\int h_i(\mathbf{q}) h_j(-\mathbf{q}) e^{-\frac{F(\mathbf{h},\mathbf{u})}{T}} \{d\mathbf{h} d\mathbf{u}\}}{\int e^{-\frac{F(\mathbf{h},\mathbf{u})}{T}} \{d\mathbf{h} d\mathbf{u}\}} = \delta_{ij} G(q)$$

$$G_0(\mathbf{k}) = \frac{T}{\varkappa k^4}$$

Interaction between in-plane and out-of-plane modes is neglected

However, such interaction dramatically change the small q behavior of $G(q)$ due to strong anharmonicity

Anomalous scaling of bending rigidity

Integrate out the in-plane modes (D=2)

$$F(\mathbf{h}) = \frac{1}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\left[\varkappa q^4 \mathbf{h}_{\mathbf{q}}\mathbf{h}_{-\mathbf{q}} + \frac{1}{4d_c}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\int \frac{d^2\mathbf{k}'}{(2\pi)^2} \times \underline{R(\mathbf{k},\mathbf{k}',\mathbf{q})(\mathbf{h}_{-\mathbf{k}}\mathbf{h}_{\mathbf{k}+\mathbf{q}})(\mathbf{h}_{\mathbf{k}'}\mathbf{h}_{-\mathbf{q}-\mathbf{k}'})}\right]$$

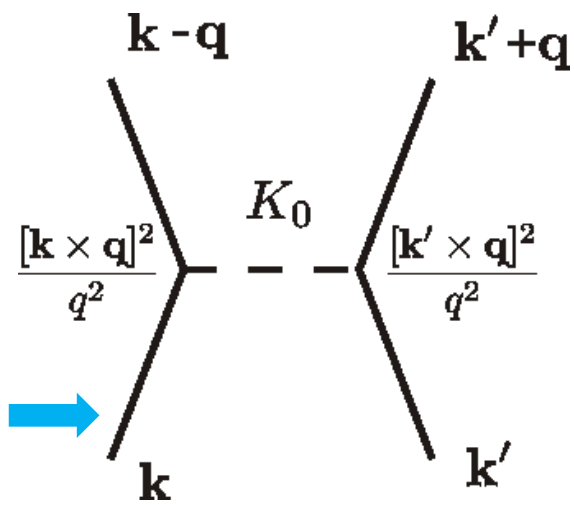
$$R(\mathbf{k},\mathbf{k}',\mathbf{q}) = K_0 \frac{[\mathbf{k}\times\mathbf{q}]^2}{q^2}\frac{[\mathbf{k}'\times\mathbf{q}]^2}{q^2}$$

$$K_0 = \frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)}$$

$$d_c = (d-D) \to \infty$$

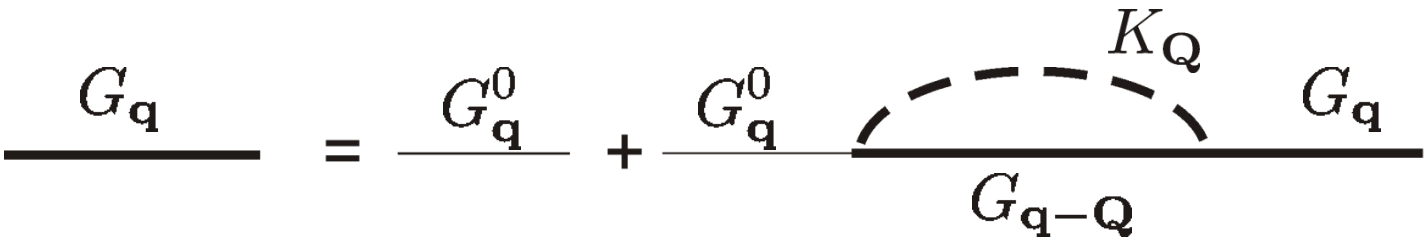
$$G^0_{\mathbf{k}} = \frac{T}{\varkappa k^4}$$

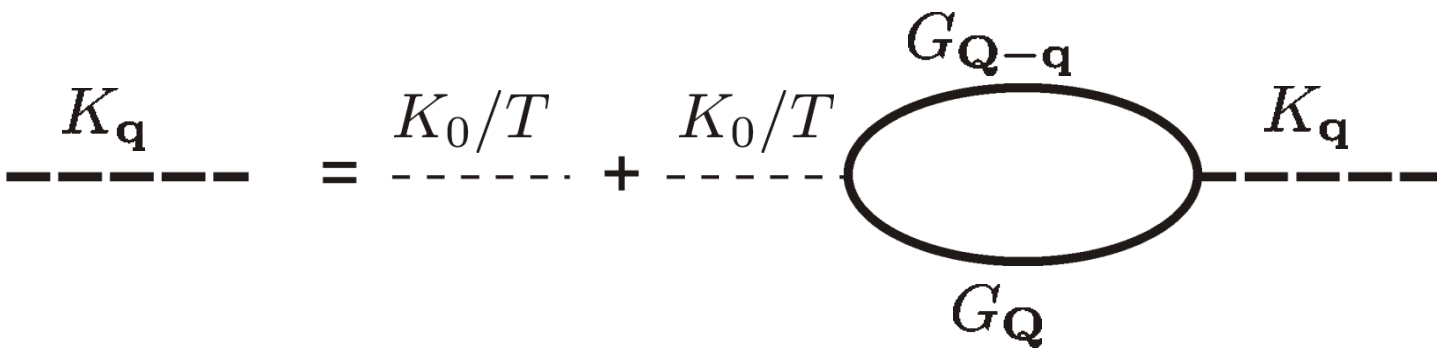
Interaction between out-of-plane modes



David, Guitter,
Europhys. Lett. (1988),

Self-Consistent Screening Approximation





$$G_{\mathbf{q}} = \frac{T}{\varkappa q^4 + \Sigma_{\mathbf{q}}}$$

$$\Sigma_{\mathbf{q}} = \frac{2T}{d_c} \int \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q} \times \mathbf{Q}]^4}{Q^4} K_{\mathbf{Q}} G_{\mathbf{q}-\mathbf{Q}}$$

Self-energy

$$K_{\mathbf{q}} = \frac{(K_0/T)}{1 + (K_0/T)\Pi_{\mathbf{q}}}$$

$$\Pi_{\mathbf{q}} = \int \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q} \times \mathbf{Q}]^4}{q^4} G_{\mathbf{Q}-\mathbf{q}} G_{\mathbf{Q}}$$

Polarization operator

$$d_c = (d - D) \to \infty$$

Universal scaling

$$\Pi^0_{\mathbf{q}} = \frac{3}{16\pi}\left(\frac{T}{\varkappa}\right)^2 \frac{1}{q^2} \to \infty, \qquad \text{for } q \to 0$$

$$q \ll q_c \;\Rightarrow\; (K_0/T)\Pi^0_{\mathbf{q}} \gg 1 \;\Rightarrow K_{\mathbf{q}} \approx \frac{1}{\Pi^0_{\mathbf{q}}} = \frac{16\pi}{3}\left(\frac{\varkappa}{T}\right)^2 q^2$$

$$q_c = \sqrt{\frac{K_0 T}{\varkappa^2}}$$

ultraviolet cutoff

$$\Sigma_{\mathbf{q}} \approx \varkappa q^4 \frac{2}{d} \ln\left(\frac{q_c}{q}\right), \qquad \text{for } q \ll q_c$$

$$\delta\varkappa = \varkappa \frac{2}{d} \ln\left(\frac{q_c}{q}\right) \Rightarrow \frac{d\varkappa}{d\Lambda} = \frac{2}{d}\varkappa$$

anharmonicity-induced increase of bending rigidity

Crumpling transition for $d \to \infty$

$$\frac{d\varkappa}{d\Lambda} = \frac{2}{d}\varkappa$$

$$\frac{d\xi^2}{d\Lambda} = -\frac{T}{4\pi\varkappa}$$

$$\boxed{\tilde{\varkappa} = \varkappa\xi^2}$$ rescaled bending rigidity

$$\frac{d\tilde{\varkappa}}{d\Lambda} = \frac{2\tilde{\varkappa}}{d} - \frac{T}{4\pi}$$

$$\tilde{\varkappa}_{cr} = \frac{d\,T}{8\pi}$$ unstable fixed point

agrees with David, Guitter, Europhys. Lett. (1988),

$$\xi_\infty^2 = \xi_0^2 \frac{\tilde{\varkappa}_0 - \tilde{\varkappa}_{cr}}{\tilde{\varkappa}_{cr}}$$

For $\tilde{\varkappa}_0 > \tilde{\varkappa}_{cr}$, membrane remains in the flat phase in the course of renormalization

$d \to \infty \;\Rightarrow\; \eta = 2/d$

$d \sim 1 \;\Rightarrow$ self consistent screening approximation (SCSA)

(similar to SCBA in the theory of disordered systems)

P. Le Doussal, L. Radzihovsky, PRL (1992)

$$G(\mathbf{q}) = \langle h_{\mathbf{q}} h^*_{\mathbf{q}} \rangle = \frac{T}{\varkappa q^4 + \Sigma(q)} \quad \Rightarrow \quad G(q) \propto \frac{1}{q^{4-\eta}}$$

$\Sigma(\mathbf{q})$ is self-energy which should be found self-consistently with the Green function

SCSA (d=3,D=2): $\eta \approx 0.82$

numerical simulations: $\eta \approx 0.7–0.8$

Effect of anharmonicity on transport properties

$$\varkappa \to \varkappa(q) \sim \varkappa \left(\frac{q_c}{q}\right)^{\eta}$$

P. Le Doussal, L. Radzihovsky, PRL (1992)

$$G_q = \langle h_{\mathbf{q}} h^*_{\mathbf{q}} \rangle = Z \frac{T}{\varkappa q^4} \left(\frac{q}{q_c}\right)^{\eta}, \quad \text{for } q \ll q_c$$

$Z \approx 3.5$,
K. V. Zakharchenko *et al*, PRB (2010)

$$q_c = \frac{\sqrt{T\Delta_c}}{\hbar v}, \quad \Delta_c = \frac{3\mu v^2(\mu+\lambda)\hbar^2}{4\pi\varkappa^2(2\mu+\lambda)} \simeq 18.7 \text{ eV}.$$

In the Dirac point: $q \sim T/\hbar v$

$$q \ll q_c \quad \longleftrightarrow \quad T \ll \Delta_c$$

For all realistic temperatures anharmonic coupling is important !!!

Quasielastic scattering by FP: effect of anharmonicity

$$V = g_1(\nabla h)^2/2 \qquad \langle h_{\mathbf{q}} h^*_{\mathbf{q}} \rangle = Z \frac{T}{\varkappa q^4} \left(\frac{q}{q_c}\right)^{\eta}$$

Drude conductivity in the Dirac point

FP

$$\frac{1}{\tau_{tr}(\epsilon)} \simeq \frac{1.4\ Z^2 g^2 T^2}{\hbar|\epsilon|} \left(\frac{|\epsilon|}{\sqrt{T\Delta_c}}\right)^{2\eta}$$

$$\sigma_{ph} \simeq \frac{e^2}{\hbar} \frac{N}{16.5\ Z^2 g^2} \left(\frac{\Delta_c}{T}\right)^{\eta}$$

Golden Rule calculation

$$g = \frac{g_1}{\sqrt{32}\varkappa} \simeq 5.3$$

$N = 4$ spin×valleys

For room temperature and $\eta = 0.72$

$$(\Delta_c/T)^{\eta} \simeq 10^2, \qquad \sigma_{ph} \approx 0.5 e^2/h$$

Electron-Electron interaction:

1) velocity relaxation ➔ additional scattering

Kashuba, PRB (2008)

$$\sigma_{ee} = \frac{e^2}{\hbar}\frac{\ln 2}{2\pi g_e^2}$$

$$g_e = \frac{g_e^0}{1 + (g_e^0/4)\ln(\Delta/T)}, \quad g_e^0 = \frac{e^2}{\hbar\kappa v_F}$$

$$\hbar/\tau_{ee} \sim N g_e^2 T, \quad \text{for } \epsilon \sim T$$

2) screening of deformation potential

E.Mariani, F.von Oppen, PRB (2010)

$$g \to \frac{g}{1 + 2\pi e^2 N \Pi(Q)/\kappa Q}$$

$\Pi(Q)$ - polarization operator

Interplay of e-ph and ee interaction

Temperature dependent coupling constants

$$G = 2NZ^2 g^2 \left(\frac{T}{\Delta_c}\right)^{\eta}, \quad G_e = g_e^2 N^2$$

$$\sigma_{\mathrm{ee+ph}} = \frac{e^2 N^2 \ln 2}{h} \Sigma(G, G_e)$$

dimensionless function

In the absence of screening

$$\Sigma(G, G_e) = \frac{1}{G + G_e}$$

Scattering by screened FP

$$\Pi(Q) = \begin{cases} \dfrac{T \ln 2}{\pi \hbar^2 v^2}, & \text{for} \quad Q \ll T/\hbar v \\ \dfrac{Q}{16 \hbar v}, & \text{for} \quad Q \gg T/\hbar v \end{cases}$$

finite T, Dirac point:
M.Schutt, P.Ostrovsky,
I.Gornyi, A.Mirlin, PRB (2011)

Dirac point: more e-h pairs ➔ stronger screening

$$Q \sim |\epsilon|/\hbar v \qquad g \to \frac{g}{1 + g_e N T/|\epsilon|} \to \frac{g|\epsilon|}{g_e N T}, \quad \text{for } \epsilon \to 0$$

coupling with phonons is fully screened in the Dirac point !!!

$$\frac{\hbar}{\tau_{tr}^{ph}(\epsilon)} \simeq \frac{Z^2 g^2 T^2}{\epsilon} \left(\frac{|\epsilon|}{\sqrt{T \Delta_c}} \right)^{2\eta} \begin{cases} \dfrac{\epsilon^2}{(g_e N T)^2}, & \text{for } \epsilon \ll g_e N T \\ 1, & \text{for } \epsilon \gg g_e N T \end{cases}$$

$$\frac{1}{\tau_{ph}} \propto |\epsilon|^{2(1+\eta)} \quad \Rightarrow \quad \sigma_{ph} \propto \int |\epsilon| \tau_{ph}(\epsilon) d\epsilon \propto \int \frac{d\epsilon}{\epsilon^{1+2\eta}}$$

divergent !!! ➔ low energies
shunt conductivity ➔ phonons are
strong but can not limit conductivity

Competition between screened FP and ee collisions

$$\frac{1}{\tau_{ph}} \propto |\epsilon|^{2(1+\eta)} \quad \Rightarrow \quad \sigma_{ph} \propto \int |\epsilon| \tau_{ph}(\epsilon) d\epsilon \propto \int \frac{d\epsilon}{\epsilon^{1+2\eta}}$$

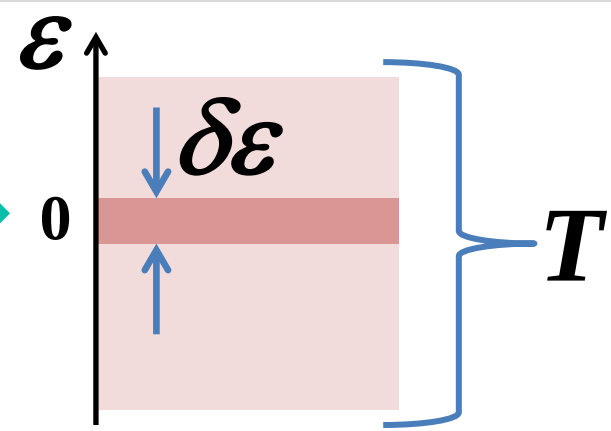
divergent !!! ➔ low energies shunt conductivity

$$\frac{\hbar}{\tau_{ee}} \propto \sqrt{|\epsilon| T}/N$$

for $\varepsilon \to 0$

Plasmon-assisted scattering
M.Schütt, P.Ostrovsky, I.Gornyi, A.Mirlin, PRB (2011)

$$\frac{1}{\tau} \propto \sqrt{\epsilon} + \epsilon^{2(1+\eta)}$$

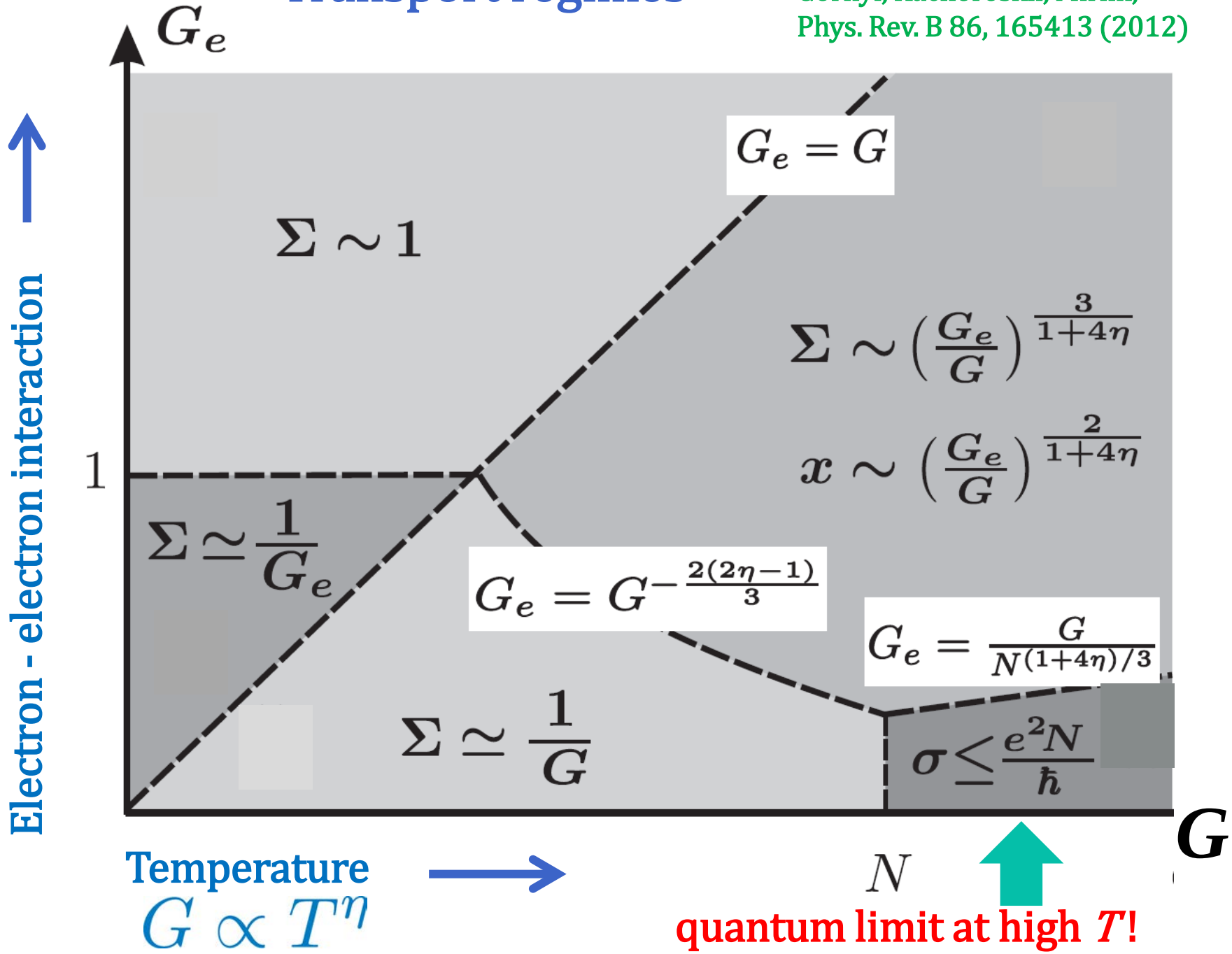


new energy scale within the temperature window

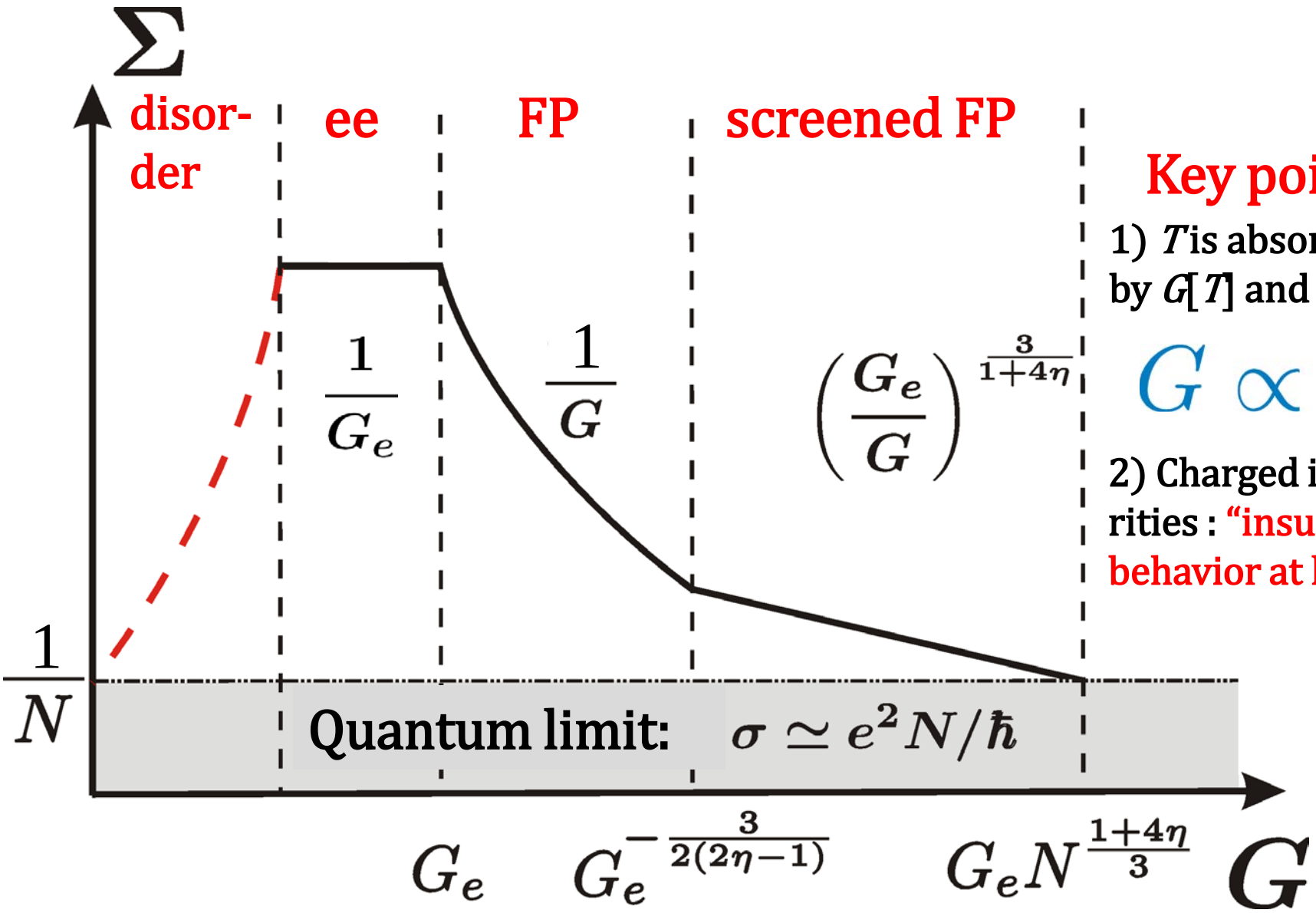
$$\frac{\delta\epsilon}{T} \simeq \left(\frac{G_e}{G}\right)^{2/1+4\eta} \ll 1 \,, \qquad \Sigma \simeq \left(\frac{G_e}{G}\right)^{3/1+4\eta}$$

Transport regimes

Gornyi, Kachoroskii, Mirlin,
Phys. Rev. B 86, 165413 (2012)



Temperature dependence of conductivity



Key points:

1) T is absorbed by $G[T]$ and $G_e[T]$

$$G \propto T^{\eta}$$

2) Charged impurities : "insulating" behavior at low T

Away from Dirac point: effect of impurities

$$\frac{1}{\tau_i(\epsilon)} \propto \frac{n_i}{|\epsilon|} \quad \Rightarrow \quad \sigma \simeq \frac{e^2 N}{\hbar} \left\langle \frac{\epsilon \tau_i(\epsilon)}{\hbar} \right\rangle_{\epsilon-\mu\sim T} \propto \frac{\mu^2 + T^2}{n_i}$$

charged impurities

without phonons

Phonons:

$$\frac{1}{\tau_{tr}(\epsilon, T)} = \frac{1}{\tau_{ph}(\epsilon, T)} + \frac{1}{\tau_i(\epsilon)}$$

$$\tau_{tr}(\epsilon, T) \approx \tau_i(\epsilon) - \tau_i^2(\mu)/\tau_{ph}(\mu, T)$$

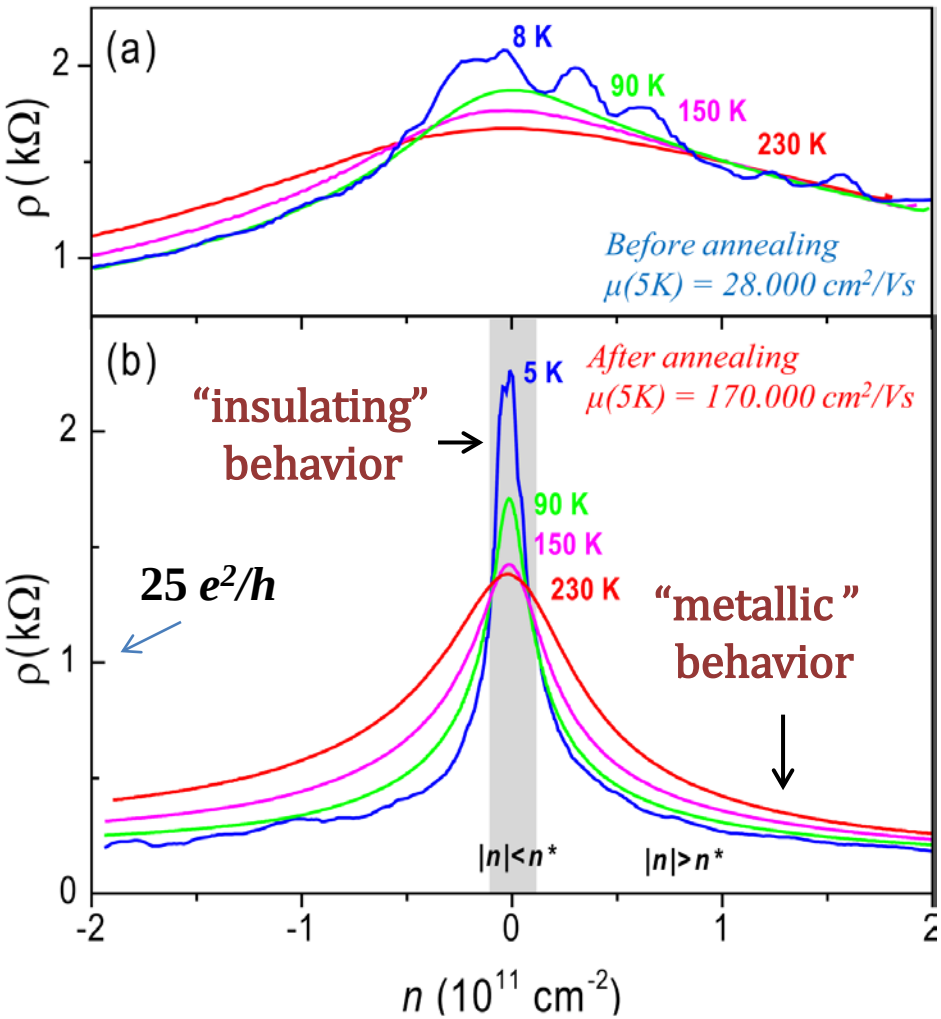
$$\sigma \propto \frac{e^2 N}{\hbar} \left\langle \frac{\epsilon \tau_{tr}(\epsilon, T)}{\hbar} \right\rangle_{\epsilon-\mu\sim T} \propto \frac{\mu^2}{n_i} + \frac{T^2}{n_i} - \frac{\mu \tau_i^2(\mu)}{\tau_{ph}(\mu, T)}$$

$$\delta\sigma \propto \frac{T^2}{n_i} - \frac{\mu^{2(1+\eta)}}{n_i^2} T^{2-\eta}$$

T– dependence is "insulating" for small μ and "metallic" for large μ

Comparison with experiment

K. Bolotin, K.Sikes, J.Hone,
H.Stormer, P.Kim, PRL (2008)



Suspended graphene (experiment):

"metallic" ↔ "insulating"
T-dependence

Suspended graphene (theory) : "metallic"↔"insulating" T-dependence

Realistic samples: disorder + Coulomb + phonons

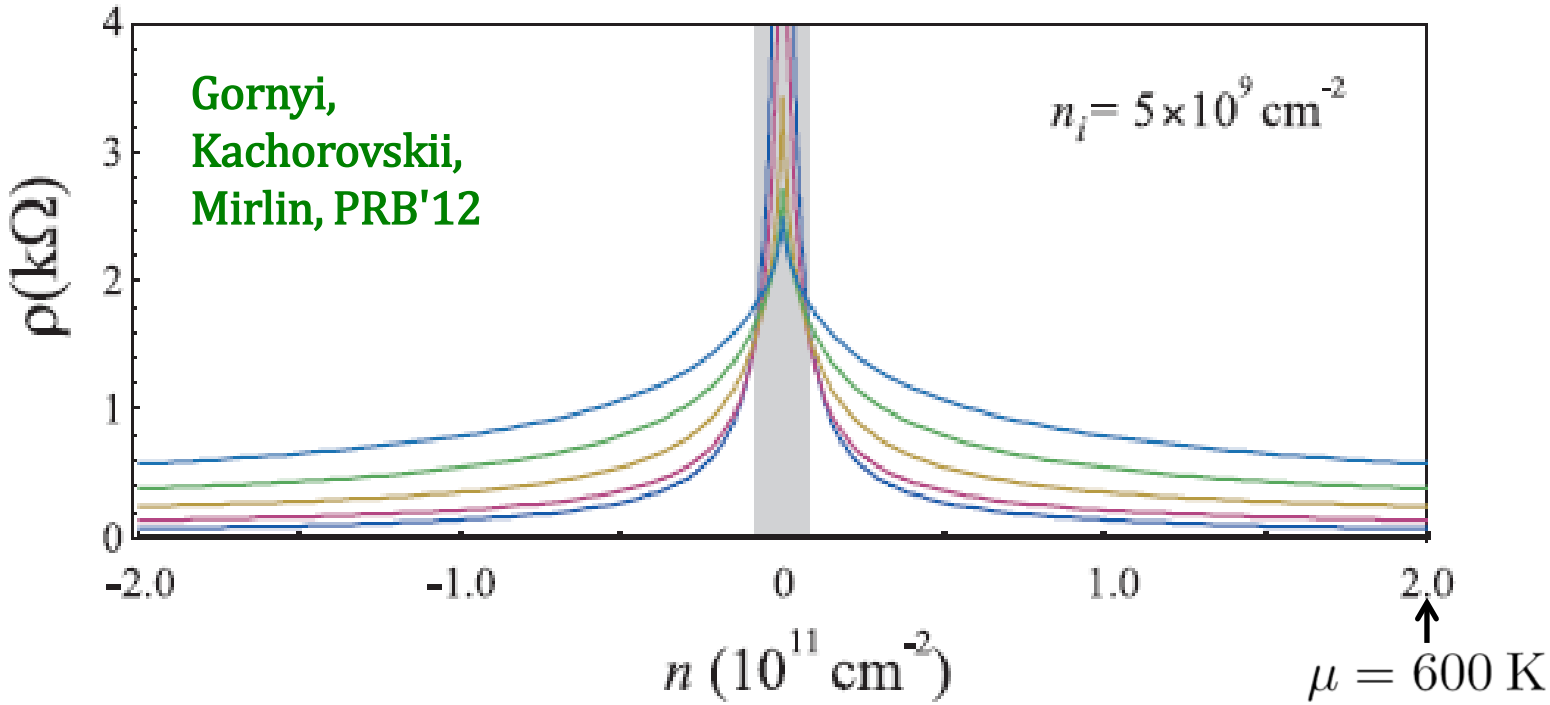
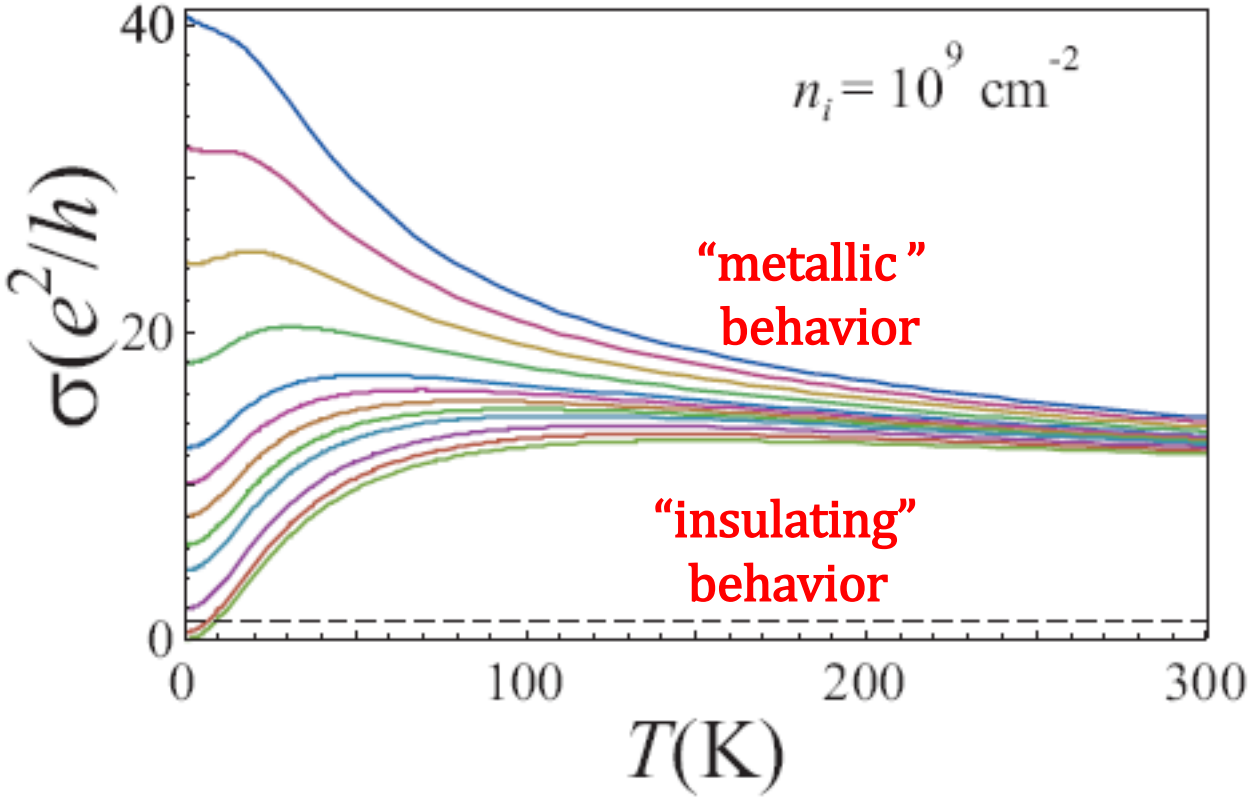


FIG. Resistivity as a function of electron concentration at $n_i = 5 \times 10^9$ cm^{-2} and different temperatures ($T/1\text{K} = 5, 40, 90, 150, 230$) increasing from the bottom to the top at large n. Within the grey area temperature dependence is "insulating", while outside this region it is "metallic".

Suspended graphene (theory) : "metallic"↔"insulating" T-dependence



Gornyi, Kachorovskii, Mirlin, PRB'12

FIG. : Conductivity at fixed impurity concentration ($n_i = 10^9$ cm^{-2}) for different values of chemical potential ($\mu/1$ K = 0, 10, 20, 30, 35, 40, 45, 50, 60, 70, 80, 90) increasing from the bottom to the top. Dashed line corresponds to SCBA limit $\sigma = 4e^2/\pi h$.

Effect of disorder on crumpling transition

$$e^{-F(\mathbf{r})/T} = \int \{d\chi_{\alpha\beta}\}\, e^{-\int d^D\mathbf{x}\left\{\frac{\varkappa d}{2T}\left[\Delta\mathbf{r} + \boldsymbol{\beta}(\mathbf{x})\right]^2 + \frac{id}{2}\chi_{\alpha\beta}(\partial_\alpha\mathbf{r}\partial_\beta\mathbf{r} - \delta_{\alpha\beta}) - \frac{Td}{4\mu}\left(\chi^2_{\alpha\beta} - \frac{\lambda}{2\mu+\lambda D}\chi^2_{\alpha\alpha}\right)\right\}}$$

random vector with the statistical weight:

$$P(\boldsymbol{\beta}) = \exp\left[-\frac{d}{2B}\int d^D x \boldsymbol{\beta}^2(\mathbf{x})\right]$$

$$\langle \ln Z\rangle_{\boldsymbol{\beta}} = \lim_{N\to 0}\left\langle \frac{Z^N - 1}{N}\right\rangle_{\boldsymbol{\beta}}$$

$$\hat{M} = \delta_{nm}\left(-\varkappa\Delta^2 + iT\partial_\alpha\chi^n_{\alpha\beta}\partial_\beta\right) + \frac{B\varkappa^2}{T}\Delta^2 \qquad n, m = 1, \ldots, N$$

$$\chi^n_{\alpha\beta} = -i\chi\delta_{\alpha\beta}$$

$$\ln\det\hat{M} = \int_0^\Lambda \frac{d^D\mathbf{k}}{(2\pi)^D}\left[(N-1)\ln\left(\varkappa k^4 + T\chi k^2\right) + \ln\left(\varkappa k^4 + T\chi k^2 - NBk^4\frac{\varkappa^2}{T}\right)\right]$$

$$F_{eff} \propto \chi(1-\xi^2) + \frac{T\chi^2}{2\mu+\lambda D} - \frac{1}{D}\int_0^\Lambda \frac{d^D\mathbf{k}}{(2\pi)^D}\left[\ln\left(\varkappa k^4 + T\chi k^2\right) - \frac{B\varkappa^2 k^2}{T(\varkappa k^2 + \chi T)}\right]$$

disorder-induced contribution

Saddle-point equations

$$\frac{\partial F_{eff}}{\partial \chi} = 0 \Rightarrow \quad 1 - \xi^2 + \chi \frac{2T}{2\mu + \lambda D} = \frac{T}{D} \int \frac{d^D k}{(2\pi)^D} \left[\frac{1}{\varkappa k^2 + \chi T} + \frac{B \varkappa^2 k^2 / T}{(\varkappa k^2 + \chi T)^2} \right]$$

$$\frac{\partial F_{eff}}{\partial \xi} = 0 \Rightarrow \quad \xi \chi = 0$$

both terms logarithmically diverge for D=2

In the flat phase: $\xi \neq 0 \Rightarrow \chi = 0$

$$\frac{d\xi^2}{d\Lambda} = -\frac{1}{4\pi} \left(\frac{T}{\varkappa} + B \right)$$

thermal fluctuations

disorder

$\xi \to 0$, for certain value of L

Flat phase is destroyed both by thermal fluctuations and by disorder

$$\Lambda = \ln (L/a)$$

RG equations

$$\frac{d\varkappa}{d\Lambda} = \frac{2}{d}\varkappa\frac{1+3B\varkappa/T+B^2\varkappa^2/T^2}{(1+2B\varkappa/T)^2}$$

$$\frac{d}{d\Lambda}\left(\frac{B\varkappa^2}{T}\right) = \frac{2}{d}\varkappa\frac{(B\varkappa/T)^3}{(1+2B\varkappa/T)^2}$$

Rescaled parameters

$$\tilde{\varkappa} = \varkappa\xi^2 \qquad F = \frac{B\varkappa^2\xi^2}{T}$$

$$\Rightarrow \quad f = \frac{F}{\tilde{\varkappa}}$$

$$\frac{df}{d\Lambda} = -\frac{2}{d}\frac{f(1+3f)}{(1+2f)^2}$$

$$\frac{d\tilde{\varkappa}}{d\Lambda} = \frac{2}{d}\tilde{\varkappa}\frac{(1+3f+f^2)}{(1+2f)^2} - \frac{T}{4\pi}$$

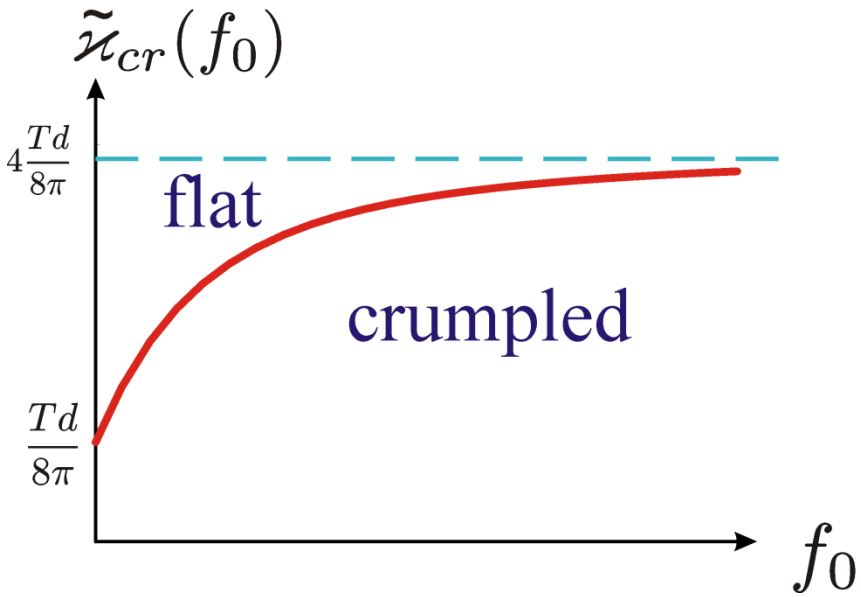
$$\frac{d\xi^2}{d\Lambda} = -\frac{\xi^2(1+f)T}{4\pi\tilde{\varkappa}}$$

$$\Lambda \to \infty \quad \Rightarrow \quad \begin{cases} f \propto \exp\left(-\frac{2}{d}\Lambda\right) \\ \tilde{\varkappa} \propto \exp\left(\frac{2}{d}\Lambda\right) \end{cases}$$

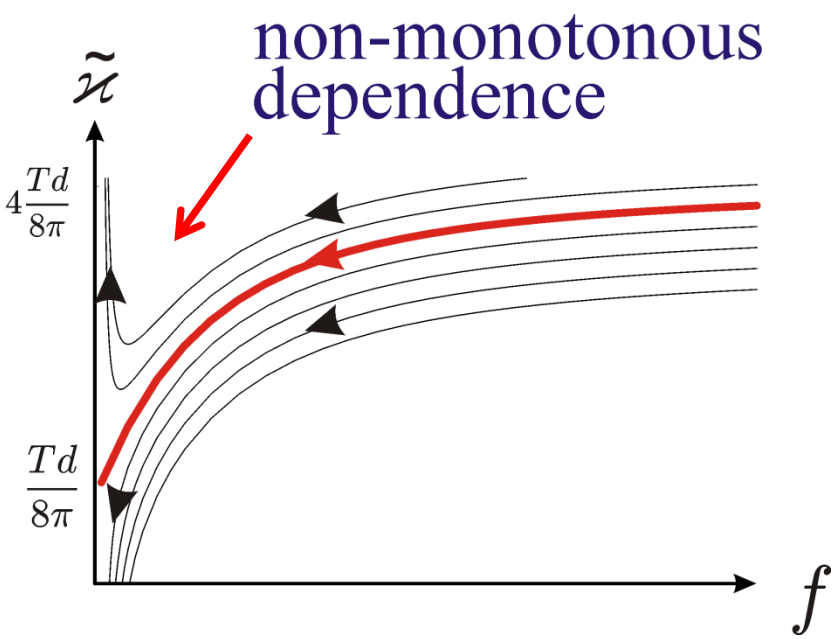
$$F = f\tilde{\varkappa} \;\longrightarrow\; \text{const}$$

Results:

Critical bending rigidity becomes disorder dependent



Non-monotonous scaling of bending rigidity



$$\tilde{\varkappa}_{cr}(f_0) = \frac{Td}{8\pi}\int_0^{f_0}\frac{df}{f}\frac{(1+2f)^2}{1+3f}\exp\left(-\int_f^{f_0}\frac{df'}{f'}\frac{1+3f'+f'^2}{1+3f'}\right)$$

Rescaled disorder strength increases exponentially and then saturates

$$\frac{F_\infty}{F_0} \sim e^{f_0/3}$$

Gornyi, Kachorovskii, Mirlin, in preparation, similar result for D=4: Morse, Lubensky, Grest, PRA 1992

$$\frac{df}{d\Lambda} = -\frac{2}{d}\frac{f(1+3f)}{(1+2f)^2} \quad \Rightarrow \quad f = f_0 - \frac{3}{2d}\ln\left(Lq_*\right), \text{ for f} \gg 1$$

characteristic scale: $L_0 \sim q_*^{-1} e^{2df_0/3}$ ripple size???

Disorder generates new correlation functions

Conventional correlation function

$$\overline{\langle h_{\mathbf{q}} h_{-\mathbf{q}}\rangle} \propto \frac{1}{q^{4-2/d}} \quad \Rightarrow \quad h_{\rm rms} \propto L^{1-1/d}$$

flat phase

Disorder-induced correlation function

$$\overline{\langle h_{\mathbf{q}}\rangle\langle h_{-\mathbf{q}}\rangle} \propto \frac{1}{q^{4-4/d}} \quad \Rightarrow \quad \tilde{h}_{\rm rms} \propto L^{1-2/d}$$

Main results

Anharmonicity crucially effects elastic and transport properties of graphene

Power law scaling of conductivity (σ) with T

"Metallic" ↔ "insulating" T-dependence of σ

Formation of ripples in disordered graphene